

## 6.4 Functions

A **relation** is a connection between two quantities. In this lesson, we will study a special type of relation called a function.

### FUNCTIONS

A function is a special type of relation in which each element of the domain is related to exactly one element of the range. In other words, each input (or x-value) should only result in one output (or y-value).

*x-values*

**Functions**

x	5	6	7
y	10	15	20

$\{(-2,-5),(0,4),(2,13),(4,22)\}$

x	11	21	31
y	3	3	3

$\{(10,10),(12,10),(14,12),(16,12)\}$

**Not Functions**

x	6	6	7
y	10	15	20

$\{(10,10),(12,10),(12,14),(12,16)\}$

x	3	3	3
y	10	15	20

*Different y's for the same x.*

### How to determine whether a relation is a function?

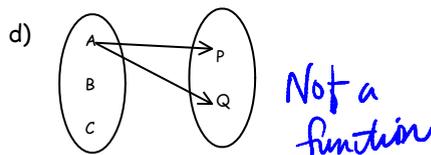
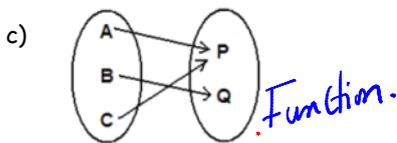
1. **Table of values and ordered pairs:** For each pair, check to see which relation has a domain value associated with more than one range. Which of the following are functions?

If one x-value produces two different y-values, then it is not a function

- a)  $(5, 8), (6, 7), (-5, 3), (2, 3), (6, 8)$       b)  $(3, 3), (2, 3), (4, 5), (-3, 2)$

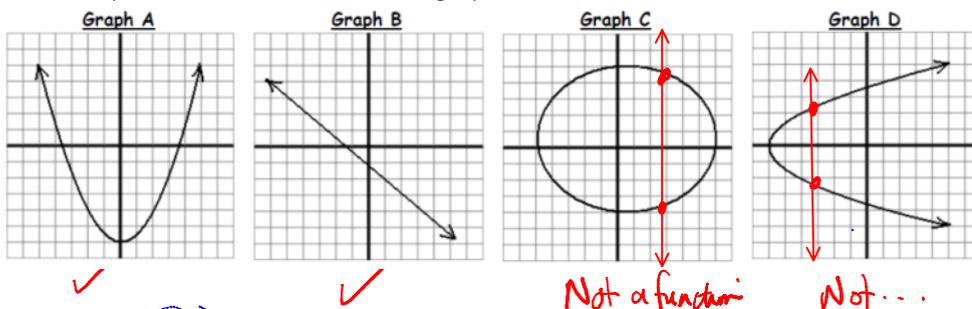
*Not a function*

*Yes, a function*



2. **Graph:** The **vertical line test** can be used on a graph of a relation to determine whether the relation is a function or not.

- If every vertical line, drawn on the domain of the relation, intersects the graph exactly once, then the relation is a function
- If any vertical line intersects the graph more than once, then it is not a function.



### Function Notation

Functions can be written using **function notation**.

$$y = 3x - 2$$

$$14 = 3(5) - 2$$

$y = 3x - 2$  can be written as  $f(x) = 3x - 2$ . 'f' is the name of the function and x is the name of the input variable (other letters can be used).

$f(x)$  replaces y in the function and is read "f of x" or "f at x"

If we chose 3 as the input, the ordered pair associated with this function would be...

replace 'x' with 3

$$f(x) = 3x - 2$$

$$f(3) = 3(3) - 2 = 7$$

$$\therefore f(3) = 7$$

Example: Determine the following if the function g is defined as  $g(x) = 5 - 3x$ .

a)  $g(4) = 5 - 3x$   
 replace x with 4  
 $5 - 3(4)$   
 $5 - 12$   
 $g(4) = -7$

b)  $g(-2) = 5 - 3x$   
 $5 - 3(-2)$   
 $= 5 + 6$   
 $g(-2) = 11$

c)  $g(\frac{2}{3}) = 5 - 3x$   
 $5 - 3(\frac{2}{3})$   
 $= 5 - 2$   
 $g(\frac{2}{3}) = 3$

d)  $g(-c) = 5 - 3x$   
 replace x with -c  
 $5 - 3(-c)$   
 $= 5 + 3c$

e)  $g(k+2) = 5 - 3x$   
 replace x  
 $5 - 3(k+2)$   
 $= 5 - 3k - 6$   
 $= -1 - 3k$

f)  $g(x+2) = 5 - 3x$   
 $5 - 3(x+2)$   
 $5 - 3x - 6$   
 $-1 - 3k$

Example: For this example, the function f is defined as  $f(x) = 2x - 3$ .

a) Find the value of x for which  $f(x) = -1$

don't know x, so find.

$$-1 = 2x - 3$$

$$-1 + 3 = 2x$$

$$\frac{2}{2} = \frac{2x}{2}$$

$$1 = x$$

$\therefore f(1) = -1$   
 found x

b) Find the value of x for which  $f(x) = 7$

$$f(x) = 2x - 3$$

$$7 = 2x - 3$$

$$7 + 3 = 2x$$

$$10 = 2x$$

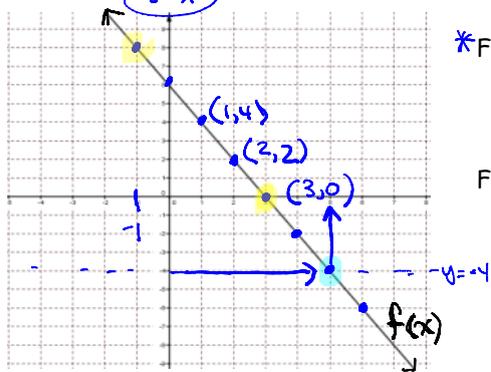
$$5 = x$$

$\therefore f(5) = 7$

Example:

Find output? y?

$f(3) = 0$   
 $x = 3$   
 $f(-1) = 8$



\* Find x if  $f(x) = 0$

don't know x?  
 $x = 3$

Find x if  $f(x) = -4$

$x = 5$

$C = \pi d$   
 $C(d) = \pi d$

Circumference is calculated by multiplying pi by the diameter. Write this formula using function notation.

$A = \pi r^2$   
 $A(r) = \pi r^2$

Assignment: p311 #1-6,8,9ab,10,11